647510 **MSC2MatC203x** **Seat No :________**

M.Sc.(Maths.) Semester - 2 (*CBCS*) Examination
March/April - 2025 (NEW COURSE)
Topology -2 (Core)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que1(A) Answer any two questions out of three. **(10)**

(1) Show that the T_1 axiom is equivalent to the condition that for each pair of points of X, each has a neighbourhood not containing the other.

(2) Let X be a T_1 space, and let A be a subset of X. Then show that $x \in X$ is a limit point of A if and only if every neighbourhood of x contains infinitely many points of A.

(3) Show that X is Hausdorff space if and only if the diagonal $\Delta = \{x \times x : x \in X\}$ is closed in $X \times X$.

Que-1(B) Answer any one question out of two. **(04)**

(1) Show that the product of two Hausdorff spaces is Hausdorff.

(2) Show that every finite point set is closed in Hausdorff space

Oue-2(A) Answer any two questions out of three. **(10)**

(1) Show that X is regular if and only if for given a point x of X and a neighbourhood U of x, there is a neighbourhood V of x such that $\bar{V} \subset U$.

(2) Show that X is normal if and only if for given a closed set A of X and an open set U containing A, there is an open set V containing A such that $\bar{V} \subset U$.

(3) Show that a regular space with countable basis is normal.

Que-2(B) Answer any one question out of two. **(04)**

(1) Show that a subspace of regular space is regular.

(2) Show that every compact Hausdorff space is regular.

Que-3(A) Answer any two questions out of three. **(10)**

(1) Let Y be a subspace of X. Then show that Y is compact if and only if each covering of Y by open subset A of X contains a finite subcollection covering of A.

(2) Let A be an open cover of the metric space (X, d). Show that if X is compact, then there is $\delta > 0$ such that for each subset of X having diameter less than δ is contained in an element of A.

(3) Sate and prove Lebesgue number lemma.

Que-3(B) Answer any one question out of two. **(04)**

(1) Show that every compact subspace of Hausdorff space is closed.

(2) State and prove Tube lemma.

Que-4(A) Answer any two questions out of three. **(10)**

(1) Let $f: X \to Y$ be continuous map from compact metric space (X, d_X) to metric space (Y, d_Y). Then show that f is uniform continuous.

(2) Let X be a metrizable space. If X is limit point compact, then show that X is sequentially compact.

(3) Let X be a metrizable space. If X is sequentially compact, then show that X is compact.

Que-4(B) Answer any one question out of two. **(04)**

(1) Let (X, d) be a metric space and $A \subset X$ be a nonempty, then show that $d(x, A) = 0$ if and only if $x \in \bar{A}$.

(2) Give an example to show that every limit point compact space is not compact.

Que-5(A) Answer any two questions out of three. **(10)**

(1) Show that a metric space (X, d) is complete, if every Cauchy sequence has convergent subsequence.

(2) Show that the Euclidean space $\mathbb{R}^k$ is complete in the square matric ρ.

(3) Let (X, d) be a metric space. Then show that there exists an isometric imbedding of X into a complete metric space.

Que-5(B) Answer any one question out of two. **(04)**

(1) Show that the closed metric subspace of a complete metric space is complete.

(2) Show that every compact metric space is complete.

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